2017年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2017

学科試験 問題 EXAMINATION QUESTIONS

(学部留学生) UNDERGRADUATE STUDENTS

数 学 (B)

MATHEMATICS (B)

注意 ☆試験時間は60分。 PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

MATHEMATICS (B)

(2017)

| Nationality | | No. | | | |
|-------------|---|-----|-------|--|--|
| Name | (Please print full name, underlining family name) | | Marks | | |

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with the correct numbers.

$$(1) \log_{10} \frac{4}{5} + 2\log_{10} 5\sqrt{5} = \boxed{}.$$

- (2) The set of all solutions of the inequality $2\cos x \sqrt{3} < 0$ with $0 \le x < 2\pi$ is the interval $\pi < x < \pi$.
- (3) If $\sqrt{2-2a} = a$, then $a = \frac{1}{a}$

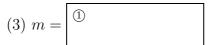
(5)
$$\sum_{k=0}^{10} \binom{10}{k} = \boxed{\qquad}$$
, where $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.

(6) Let x, y, z be natural numbers with x < y < z. If $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, then $(x, y, z) = \left(\boxed{\textcircled{1}} \right)$, $\boxed{\textcircled{2}} \right)$.

- **2.** Suppose that the parabola $y = x^2$ and a line l have two intersection points (a, a^2) and (b, b^2) (a < b). Fill in the blanks with the answers to the following questions.
 - (1) Let S be the area of the region bounded by the parabola $y = x^2$ and the line l. Express S in terms of a and b.
 - (2) When the line l is perpendicular to the tangent line to the parabola $y = x^2$ at the point (b, b^2) , express a in terms of b.
 - (3) When the condition in (2) holds, calculate the minimum m of the area S, and the value of b at which S attains its minimum m.







$$b = \bigcirc{2}$$

- **3.** Consider the solid body B formed by the intersection of the two cylinders $x^2 + z^2 \le r^2$, $y^2 + z^2 \le r^2$ (r > 0) in xyz-space. Fill in the blanks with the answers to the following questions.
 - (1) Express the area of the cross section of B cut by the plane z = t $(-r \le t \le r)$ in terms of r, t.
 - (2) Express the volume of B in terms of r.

