2018 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT（MEXT）SCHOLARSHIPS 2018

学科試験　問題

EXAMINATION QUESTIONS

（学部留学生）

UNDERGRADUATE STUDENTS

数　　学（Ａ）

MATHEMATICS（A）

注意　☆試験時間は６０分。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	

1. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) The number of digits of 7^{2677} is [1-1] and the last digit of it is [1-2], where $\log_{10} 3 = 0.4771$, $\log_{10} 7 = 0.8451$.

(2) Simplify $\dfrac{1}{1+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + \dfrac{1}{\sqrt{4}+\sqrt{5}}$ as briefly as possible.

The result is [1-3].

(3) Assume that $0 < \theta < \dfrac{\pi}{4}$. If $\sin 2\theta = \dfrac{1}{4}$, then $\dfrac{\sin\theta + \cos\theta}{-\sin\theta + \cos\theta} =$ [1-4].

(4) Let P_1, P_2, P_3, P_4, P_5, and P_6 be the vertices of a regular hexagon in anticlockwise order. We throw a fair dice three times and denote the scores shown on the dice as the ordered triple (i, j, k). In this case, the probability that the three points P_i, P_j, P_k make a triangle is $\dfrac{\text{[1-5]}}{9}$.

(5) For the equation $4^x - 2^x - 12 = 0$, the real solution is $x =$ [1-6].

(6) For a pyramid $OABC$, the centroids of the triangles OAB, OBC, and OCA are F, G, and H, respectively. For the centroid P of the triangle FGH, the vector $\vec{OP}$ is given by

$$\vec{OP} = \frac{2}{\text{[1-7]}} \left(\vec{OA} + \vec{OB} + \vec{OC} \right).$$

(7) Let a point O be the origin of xy-coordinate plane. We define four points $A(1,\ 0)$, $B(1,\ 1)$, $C(2,\ 1)$, $D(3,\ 1)$ on the plane. Let us start from C, go through a point on line OA, go through a point on line OB, and reach D with the minimum length of the path. In this path, the point on the line OA is ([1-8] , [1-9]), that on the line OB is ([1-10] , [1-11]), the length of the path is [1-12] .

(8) Assume that integers m and n satisfy $2|m|+3|n-1| \leq 7$. $m+n$ is maximum when $(m,\ n) = \left(3, \boxed{[1\text{-}13]}\right)$, $\left(\boxed{[1\text{-}14]}, \boxed{[1\text{-}15]}\right)$ and its maximum value is [1-16] .

(9) If a quadratic function $f(x)$ is maximum at $x = 1$ with the maximum value 5, and satisfies $f(-2) = -22$, it is given by

$$f(x) = \boxed{[1\text{-}17]}\,x^2 + \boxed{[1\text{-}18]}\,x + \boxed{[1\text{-}19]}.$$

(10) When integers k and n satisfy $1 \leq k \leq n$, we have

$$\sum_{l=k}^{n} 2^l = 2^{\boxed{[1\text{-}20]}} - 2^{\boxed{[1\text{-}21]}}.$$

Therefore, it follows that

$$\sum_{k=1}^{n} k2^k = \sum_{k=1}^{n}\sum_{l=k}^{n} 2^l = \left(\boxed{[1\text{-}22]}\right) 2^{\boxed{[1\text{-}23]}} + 2.$$

(11) A decimal number 123456 is shown by a ternary (base 3) number [1-24] .

(Describe only the value of the ternary number without describing a notation that indicates a ternary numeral system.)

2. For a cubic function $f(x) = x^3 - 3\,a\,x^2 + 3\,b\,x - 2$, answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) If $x = 1, 3$ are the extreme points of $f(x)$, then $a =$ [2-1] and $b =$ [2-2]. In this case, the solutions of $f(x) = 0$ can be arranged as [2-3] $<$ [2-4] $<$ [2-5] in increasing order.

(2) Assume that $a = b$. If the function $f(x)$ is monotonously increasing, then [2-6] $\leq a \leq$ [2-7].

3. In xyz-coordinate system, we define a solid A by

$$\frac{1}{9}x^2 + \frac{1}{4}y^2 \leq z^4 \quad (0 \leq z \leq 1).$$

Fill in your responses in the corresponding boxes on the answer sheet.

(1) We define a solid B by

$$x^2 + y^2 \leq z^4 \quad (0 \leq z \leq 1)$$

The volume of the solid B is [3-1].

(2) The solid A is given by elongating the solid B [3-2] times in the x-axis direction and [3-3] times in the y-axis direction.

(3) The volume of the solid A is [3-4] times as large as that of the solid B.

(4) The volume of the solid A is [3-5].