QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MEXT) SCHOLARSHIPS 2018

EXAMINATION QUESTIONS

SPECIALIZED TRAINING COLLEGE STUDENTS

MATHEMATICS

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.
1. Fill in the following blanks with the correct answers.

(1) \( \log_3 0.008 = \boxed{1} \), \( (\sqrt[3]{16})^3 = \boxed{2} \).

(2) \( \sin 75^\circ + \sin 120^\circ - \cos 150^\circ + \cos 165^\circ = \boxed{\text{answer}} \).

(3) \( \frac{1}{3 \cdot 6} + \frac{1}{6 \cdot 9} + \frac{1}{9 \cdot 12} + \frac{1}{12 \cdot 15} = \boxed{\text{answer}} \).

(4) The number of integers \( x \) that satisfy the following inequalities

\(-x < x^2 < 6\)

is \( \boxed{\text{answer}} \).

(5) Among four-digit integers where digits are all different numerals, the total possible number of integers that are greater than or equal to 5000 is \( \boxed{\text{answer}} \).

(6) When \( \vec{a} + \vec{b} + \vec{c} = \vec{0} \) and \( |\vec{a}| = |\vec{b}| = |\vec{c}| = 1 \), then the degree measure of the angle between \( \vec{a} \) and \( \vec{b} \) is \( \boxed{\text{degree}} \) and \( |\vec{a} - \vec{b}| = \boxed{\text{distance}} \).

(7) In the progression 3, 4, 6, 10, 18, \ldots , the numeral of the 8th term is \( \boxed{\text{numeral}} \), and the number of term that is 1026 is \( \boxed{\text{number}} \).

(8) Let \( f(x) = x^2 - 4x + 1 \).

(i) \( f(-2) = \boxed{\text{value}} \).

(ii) If \( f(x) = 0 \), \( x = \boxed{\text{value}} \) or \( x = \boxed{\text{value}} \). (\( 1 < 2 \))

(iii) The area bounded by the parabola \( y = f(x) \) and the \( x \)-axis is \( \boxed{\text{area}} \).

(9) In a space with a coordinate system, there are three points \( A (0,1,1) \), \( B (-1,-1,2) \) and \( C (2,3,1) \). The area of \( \triangle ABC \) is \( \boxed{\text{area}} \).
2. A quadrangle ABCD which is inscribed in a circle on a plane satisfies $2AB = AC, \ BC = \sqrt{3}, \ BD = DC$ and $\angle BAC = 60^\circ$.

Fill in the following blanks with the correct numbers.

(1) The radius of the circumscribed circle of the quadrangle ABCD = ____. 

(2) $AC =$ ____. 

(3) $\angle BDC =$ ____°. 

(4) The area of $\triangle BDC =$ ____. 

(5) The scalar product of two vectors $\overrightarrow{DC} \cdot \overrightarrow{CA} =$ ____. 

3. On the plane $xy$, the graph of the parabola $y = ax^2 + bx + c$ is shown in the figure below. Judge whether the following expressions are larger than or smaller than zero.

Fill in the blanks with the correct marks: $>$ or $<$.

(1) $a \quad 0$ \hspace{2cm} (2) $4ac - b^2 \quad 0$ 

(3) $a + b + c \quad 0$ \hspace{2cm} (4) $4a - 2b + c \quad 0$ 

(5) $\frac{c}{a} \quad 0$ \hspace{2cm} (6) $\frac{b}{a} \quad 0$ 

(7) $b + 4a \quad 0$ \hspace{2cm} (8) $2a + b \quad 0$