MATHEMATICS (B)                                                   (2015)
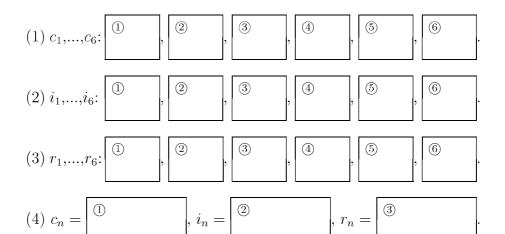
| Nationality | | No. | | | |
|---|---|---|---|---|---|
| Name | (Please print full name, underlining family name) | | | Marks | |

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

**1.** Fill in the blanks with the correct numbers.

(1) If the equation $\log_{10}(ax)\log_{10}(bx) + 1 = 0$ with $a > 0$, $b > 0$ constants has a solution $x > 0$, it follows that $\dfrac{b}{a} \geq \boxed{①}$

or $\boxed{②} \geq \dfrac{b}{a} > \boxed{③}$ .

(2) If $\cos\theta = \sqrt{\dfrac{1}{2} + \dfrac{1}{2\sqrt{2}}}$ and $\sin\theta = -\sqrt{\dfrac{1}{2} - \dfrac{1}{2\sqrt{2}}}$ with $0 \leq \theta < 2\pi$, it follows that $2\theta = \boxed{\phantom{xxxxx}} \pi$.

(3) If $y = \log_2(x + \sqrt{x^2 + 1})$, then $2^y - 2^{-y} = \boxed{\phantom{xxxxx}} x$.

(4) The function $f(x) = \log_2(\log_3(\log_2(\log_3(\log_2 x))))$ has the interval $x > \boxed{\phantom{xxxx}}$ as its maximum domain on real numbers.

(5) The total number of subtractions that result in 11111 remaining after a four-digit number has been subtracted from a five-digit number and the digits 1 through 9 have all been used is $\boxed{\phantom{xxx}}$ .

**2.** Draw a circle C on a plane. Put $n$ distinct points on the circumference of C. Join any two of the $n$ points with a chord. Suppose that no three chords intersect at any one point inside C. Let $c_n$ be the number of such chords, $i_n$ that of intersections of the chords inside C, and $r_n$ that of regions in C bounded by an arc and/or some chords. Fill in the blanks with the answers to the following questions.

(1) Find $c_1, c_2, c_3, c_4, c_5, c_6$ and write their values in this order.

(2) Find $i_1, i_2, i_3, i_4, i_5, i_6$ and write their values in this order.

(3) Find $r_1, r_2, r_3, r_4, r_5, r_6$ and write their values in this order.

(4) Express $c_n$, $i_n$, $r_n$ with binomial coefficients in terms of $n$.
(For binomial coefficients $\binom{m}{k}$, note that $\binom{m}{k} = 0$ if $m < k$.)

(1) $c_1,...,c_6$:  ① [ ]  , ② [ ]  , ③ [ ]  , ④ [ ]  , ⑤ [ ]  , ⑥ [ ]  .

(2) $i_1,...,i_6$:  ① [ ]  , ② [ ]  , ③ [ ]  , ④ [ ]  , ⑤ [ ]  , ⑥ [ ]  .

(3) $r_1,...,r_6$:  ① [ ]  , ② [ ]  , ③ [ ]  , ④ [ ]  , ⑤ [ ]  , ⑥ [ ]  .

(4) $c_n =$ ① [ ]  , $i_n =$ ② [ ]  , $r_n =$ ③ [ ]  .

**3.** Let $C$ be the plane curve $y = f(x)$ defined by the cubic function $f(x) = x^3 - 4x^2 + ax + b$ with $a$, $b$ real constants. Fill in the blanks with the answers to the following questions.

(1) When $C$ is tangent to the $x$-axis at $x = 3$, what are $a$ and $b$?

(2) When (1) holds, find all $x$ such that $C$ has points in common with the $x$-axis.

(3) When (1) holds, calculate the area $S$ of the limited region bounded by $C$ and the $x$-axis.

(1) $a = $ ① $\boxed{\phantom{xxxxxxx}}$ , $b = $ ② $\boxed{\phantom{xxxxxxx}}$ .

(2) $x = \boxed{\phantom{xxxxxxx}}$ .

(3) $S = \boxed{\phantom{xxxxxxx}}$ .