2017 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT（MONBUKAGAKUSHO）SCHOLARSHIPS 2017

学科試験　問題

EXAMINATION QUESTIONS

（学部留学生）

UNDERGRADUATE STUDENTS

数　　学（A）

MATHEMATICS（A）

注意　☆試験時間は６０分。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

MATHEMATICS(A) (2017)

Nationality		No.			Marks	
Name	(Please print full name, underlining family name)					

1. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) If the graph of a quadratic function $y = f(x)$ is symmetric with respect to $x = 1$ and goes through the points $(0, 2)$ and $(3, 5)$, then the function is

$$f(x) = \boxed{[1\text{-}1]}\,x^2 + \boxed{[1\text{-}2]}\,x + \boxed{[1\text{-}3]}.$$

(2) There is a triangle with the sides $5, 7, 8$. The area of the triangle is $\boxed{[1\text{-}4]}$ and the radius of inscribed circle in the triangle is $\boxed{[1\text{-}5]}$.

(3) If the natural numbers are divided into groups with n elements for n-th group as follows:

$$1 \mid 2, 3 \mid 4, 5, 6 \mid 7, 8, 9, 10 \mid 11, 12, \ldots,$$

then the number 2675 is No. $\boxed{[1\text{-}6]}$ in the $\boxed{[1\text{-}7]}$ group.

(4) For the natural number n, it holds that

$$\frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \cdots + \frac{1}{(3n-1)(3n+2)} = \frac{\boxed{[1\text{-}8]}}{\boxed{[1\text{-}9]}}.$$

(5) There are three different sizes of red balls and different three sizes of white balls. If those six balls are lined up, the number of permutations that at least one ball at the end is red is $\boxed{\text{[1-10]}}$. If those six balls are arranged circularly and alternately by color, the number of permutations is $\boxed{\text{[1-11]}}$.

(6) Two solutions of a quadratic equation $x^2 - 3x - 1 = 0$ are

$$x_1 = \frac{\boxed{\text{[1-12]}} + \sqrt{\boxed{\text{[1-13]}}}}{2}, \quad x_2 = \frac{\boxed{\text{[1-12]}} - \sqrt{\boxed{\text{[1-13]}}}}{2}.$$

Here, integers to satisfy $m < x_1 < m+1$ and $n < x_2 < n+1$ are $m = \boxed{\text{[1-14]}}$ and $n = \boxed{\text{[1-15]}}$.

(7) To solve an inequality $\sin 2x > \sqrt{2}\cos\left(x + \frac{\pi}{4}\right) + \frac{1}{2}$ within the domain of $0 \leq x < 2\pi$, $a = \sin x$ and $b = \cos x$ are introduced to rewrite the inequality into

$$\boxed{\text{[1-16]}}\,ab + \boxed{\text{[1-17]}}\,a + \boxed{\text{[1-18]}}\,b - 1 > 0.$$

By factorizing the left-hand side, the ranges of x to satisfy the inequality are obtained as

$$\boxed{\text{[1-19]}} < x < \boxed{\text{[1-20]}} \text{ or } \boxed{\text{[1-21]}} < x < \boxed{\text{[1-22]}}.$$

(Note: $\boxed{\text{[1-19]}}$ should be smaller than $\boxed{\text{[1-21]}}$.)

(8) $x^4 - 8x^3 + 14x^2 + 8x - 1$ divided by $x^2 - 5x - 2$ results in the quotient $x^2 + \boxed{\text{[1-23]}}\,x + \boxed{\text{[1-24]}}$ and the remainder $\boxed{\text{[1-25]}}\,x + \boxed{\text{[1-26]}}$.

(9) The graph $y = \log_2 x$ is translated $\boxed{\text{[1-27]}}$ units in x and $\boxed{\text{[1-28]}}$ units in y, then the translated graph is $y = \log_2\left(\frac{x}{2} + 3\right)$. Then these two graphs cross at $\left(\boxed{\text{[1-29]}}, 1 + \log_2 \boxed{\text{[1-30]}}\right)$.

(10) If two vectors $\vec{a}$ and $\vec{b}$ satisfy $|\vec{a}| = |\vec{b}| = |\vec{a} + \vec{b}| = 1$, then $\vec{a} \cdot \vec{b} = \boxed{\text{[1-31]}}$, $|2\vec{a} + \vec{b}| = \boxed{\text{[1-32]}}$, and the angle between $2\vec{a} + \vec{b}$ and $\vec{b}$ is $\boxed{\text{[1-33]}}$ degrees.

2. The following table indicates scores in Subject A and Subject B for eight students.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
Subject A	c	a	b	a	b	a	b	c
Subject B	b	b	a	a	b	a	b	a

Assume that these scores a, b, c are different and positive integers and that the sample means of Subject A and Subject B are equal. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) When the score c is represented by the scores a, b, we have $c = \boxed{\text{[2-1]}}$.

(2) The sample mean of Subject A is $\boxed{\text{[2-2]}}$ by using the score c.

(3) Let s_A^2 and s_B^2 be the sample variance of Subject A and Subject B, respectively. Then the ratio s_A^2/s_B^2 is $\boxed{\text{[2-3]}}$, so that s_A^2 $\boxed{\text{[2-4]}}$ s_B^2 with respect to the magnitude relationship of the sample variances.

3. For real number x, functions $f(x), g(x)$ are differentiable with respect to x and satisfy the following conditions:

$$\frac{d}{dx}\{f(x)+g(x)\} = 2, \quad \frac{d}{dx}\left\{f(x)^2+g(x)^2\right\} = 4x,$$

where $f(1)=2$ and $g(1)=0$. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) The sum of $f(x)$ and $g(x)$ is as follows:

$$f(x)+g(x) = \boxed{\text{[3-1]}}.$$

(2) The product of $f(x)$ and $g(x)$ is as follows:

$$f(x)\,g(x) = \boxed{\text{[3-2]}}.$$

(3) Two functions are

$$f(x) = \boxed{\text{[3-3]}}, \quad g(x) = \boxed{\text{[3-4]}}.$$