2017 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT（MONBUKAGAKUSHO）SCHOLARSHIPS 2017

学科試験　問題

EXAMINATION QUESTIONS

（学部留学生）

UNDERGRADUATE STUDENTS

数　　学（B）

MATHEMATICS（B）

注意　☆試験時間は６０分。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

MATHEMATICS (B) (2017)

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with the correct numbers.

(1) $\log_{10}\dfrac{4}{5} + 2\log_{10} 5\sqrt{5} =$ $\boxed{}$.

(2) The set of all solutions of the inequality $2\cos x - \sqrt{3} < 0$ with $0 \le x < 2\pi$ is the interval $\boxed{①}\,\pi < x < \boxed{②}\,\pi$.

(3) If $\sqrt{2-2a} = a$, then $a =$ $\boxed{}$.

(4) The line through two points $(-1, 0)$, $(0, t)$ intersects with the unit circle $x^2 + y^2 = 1$ in one point (a, b) away from the point $(-1, 0)$.
Then, $a = \dfrac{\boxed{①} + \boxed{②}\,t + \boxed{③}\,t^2}{1+t^2}$.

(5) $\displaystyle\sum_{k=0}^{10} \binom{10}{k} =$ $\boxed{}$, where $\displaystyle\binom{n}{k} = \frac{n!}{(n-k)!k!}$.

(6) Let x, y, z be natural numbers with $x < y < z$. If $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 1$, then $(x, y, z) = \left(\boxed{①}, \boxed{②}, \boxed{③}\right)$.

2. Suppose that the parabola $y = x^2$ and a line l have two intersection points (a, a^2) and (b, b^2) $(a < b)$. Fill in the blanks with the answers to the following questions.

(1) Let S be the area of the region bounded by the parabola $y = x^2$ and the line l. Express S in terms of a and b.

(2) When the line l is perpendicular to the tangent line to the parabola $y = x^2$ at the point (b, b^2), express a in terms of b.

(3) When the condition in (2) holds, calculate the minimum m of the area S, and the value of b at which S attains its minimum m.

(1) $S =$ ☐

(2) $a =$ ☐

(3) $m =$ ① ☐ $b =$ ② ☐

3. Consider the solid body B formed by the intersection of the two cylinders $x^2 + z^2 \leq r^2$, $y^2 + z^2 \leq r^2$ $(r > 0)$ in xyz-space. Fill in the blanks with the answers to the following questions.

(1) Express the area of the cross section of B cut by the plane $z = t$ $(-r \leq t \leq r)$ in terms of r, t.

(2) Express the volume of B in terms of r.

(1) ☐

(2) ☐