2023 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR THE JAPANESE GOVERNMENT (MEXT) SCHOLARSHIP 2023

学科試験　　問題

EXAMINATION QUESTIONS

（学部留学生（文科系専攻））

UNDERGRADUATE STUDENTS

数　　学　（A）

MATHEMATICS(A)

注意　☆試験時間は **60 分**。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

MATHEMATICS (A) (2023)

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	

1. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) Let a, b, and c be real numbers. If $a+b+c=2$, $ab+bc+ca=-5$, and $abc=-6$, then the value of $a^3+b^3+c^3$ is $\boxed{[1\text{-}1]}$.

(2) Let $0^\circ \leq \theta \leq 90^\circ$. If $\sin 2\theta = \dfrac{1}{2}$, then the value of $\sin\theta + \cos\theta$ is $\boxed{[1\text{-}2]}$. Rationalize the denominator of the fraction in your answer.

(3) For the sequence $\{a_n\}$ defined by $a_1 = 2$, $a_{n+1} = 2a_n - 1$ $(n = 1, 2, \dots)$, the value of $\displaystyle\sum_{n=1}^{10} a_n$ is $\boxed{[1\text{-}3]}$.

(4) Let $0 \leq x \leq 2$ and $f(x) = 8^x - 3 \cdot 4^x + 4$. The function $f(x)$ takes a minimum $\boxed{[1\text{-}4]}$ at $x = \boxed{[1\text{-}5]}$ and a maximum $\boxed{[1\text{-}6]}$ at $x = \boxed{[1\text{-}7]}$.

(5) The value of the following formula is $\boxed{[1\text{-}8]}$.

$$\sum_{k=1}^{19} \cos\left(\frac{\pi k}{10}\right).$$

(6) The third largest common divisor of 22392 and 36216 is $\boxed{[1\text{-}9]}$.

(7) We assume that the x, y coordinates of two points P and Q are (1, 2) and (2, 5), respectively. Let R be the point such that R and Q are point-symmetric with respect to P. The x, y-coordinate of R is $\boxed{[1\text{-}10]}$. The graph that describes a set of points that are point-symmetric to the points in graph $y = x^3 + 2$ with respect to P is $y = \boxed{[1\text{-}11]}$.

(8) In a box, there are nine cards numbered from 1 to 9, one digit on each card. We take three cards from the box without returning them to the box. We construct a three-digit number in which the numbers written on the first, second, and third card are assigned as the first, second, and third digit, respectively. The probabilities that the three-digit number is a multiple of 3, that of 4, and that of 5 are $\boxed{[1\text{-}12]}$, $\boxed{[1\text{-}13]}$, and $\boxed{[1\text{-}14]}$, respectively.

2. Consider the function $f(x) = |x^2 - 4x + 3| + 2x$. Let $1 < a < 3$ and ℓ_a be a line tangent to graph $y = f(x)$ at $(a, f(a))$.

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet. Each answer should be an integer. Fractions should be simplified as much as possible and the denominators should be positive integers.

(1) $\ell_a : y = \left(\boxed{[2\text{-}1]}\, a + \boxed{[2\text{-}2]} \right) x + \boxed{[2\text{-}3]}\, a^2 + \boxed{[2\text{-}4]}$

(2) Let $S(a)$ be the area of the region enclosed by line ℓ_a and graph $y = f(x)$. At $a = \boxed{[2\text{-}5]}$, $S(a)$ takes the minimum of $\dfrac{\boxed{[2\text{-}6]}\sqrt{2}}{3} - \dfrac{\boxed{[2\text{-}7]}}{\boxed{[2\text{-}8]}}$.

3. For a pentagon ABCDE inscribed in a circle, we assume that the lengths of four edges AB, BC, CD, and DE are $-1+\sqrt{3}$, $\sqrt{2}$, 1, and 2, respectively and the angle of $\angle$ABC is $135°$. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet. Each of them should be an integer or a fraction. Fractions should be simplified as much as possible and the denominators should be positive integers. Note that some of answers may be zero or negative.

(1) The length of edge AC is $\boxed{\text{[3-1]}} + \boxed{\text{[3-2]}}\sqrt{6}$.

(2) The radius of the circle circumscribing pentagon ABCDE is $\boxed{\text{[3-3]}} + \boxed{\text{[3-4]}}\sqrt{2}$.

(3) Let θ be the angle of $\angle$ACD. $\cos\theta = \boxed{\text{[3-5]}} + \boxed{\text{[3-6]}}\sqrt{7}$.

(4) The length of edge EA is $\boxed{\text{[3-7]}} + \boxed{\text{[3-8]}}\sqrt{7}$.