

2023 年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR THE JAPANESE
GOVERNMENT (MEXT) SCHOLARSHIP 2023

学科試験 問題

EXAMINATION QUESTIONS

(学部留学生 (理科系専攻))

UNDERGRADUATE STUDENTS

数 学 (B)

MATHEMATICS (B)

注意 ☆試験時間は **60 分**。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with your answers.

- (1) The solution of the equation $\log_{81} X = \frac{5}{8}$ is $X = \boxed{}$.
- (2) Let O be the center of the circumscribed circle of a triangle ABC. If $\angle BAC = \frac{11}{12}\pi$ and $AO = 3$, then the area of triangle BOC is $\boxed{}$.
- (3) Consider a rectangle with vertices marked as A, B, C and D consecutively in the counterclockwise direction. Let P be a point between B and C, and Q be the intersection of AC and DP. If $AB = 1$, $\angle DAC = \frac{\pi}{6}$ and $\angle AQP = \frac{5}{12}\pi$, then the length of AD is $\boxed{\textcircled{1}}$ and that of PD is $\boxed{\textcircled{2}}$.
- (4) When the curve C defined by $y = x^3 - 3x^2 + 1$ and the line l meet at points $(3, 1)$ and $(-1, -3)$, the coordinates of the other intersection of C and l is $\boxed{}$.
- (5) For a real number a with $-\frac{3}{2} \leq a \leq \frac{3}{2}$, the minimum value of $\frac{1}{4 - a^3}$ is $\boxed{}$.
- (6) Consider the vectors $\vec{a} = (x, -4)$, $\vec{b} = (6, x - 7)$, and $\vec{c} = (3, -5)$. Vectors $\vec{a}$ and $\vec{b} + \vec{c}$ are parallel if $x = \boxed{\textcircled{1}}$. In this case, the vector $\vec{d} = (3, \boxed{\textcircled{2}})$ is orthogonal to $\vec{a}$.

2.

Please answer ③ and ④ with “positive” or “negative” and in ⑨, ⑩ and ⑪ with “increasing” or “decreasing”.
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Consider the function $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x - \frac{4}{3}$.

$f(x)$ can be factorized into

$$f(x) = \frac{1}{3}(x - \boxed{\text{①}})^2(x - \boxed{\text{②}}).$$

Hence $f(x)$ is $\boxed{\text{③}}$ if $x < \boxed{\text{①}}$, and $\boxed{\text{④}}$ if $x > \boxed{\text{②}}$.

The derivative of $f(x)$ is

$$f'(x) = x^2 - \boxed{\text{⑤}}x + \boxed{\text{⑥}},$$

and this can be factorized into

$$(x - \boxed{\text{⑦}})(x - \boxed{\text{⑧}}),$$

with $\boxed{\text{⑦}} < \boxed{\text{⑧}}$.

Thus $f(x)$ is $\boxed{\text{⑨}}$ if $x < \boxed{\text{⑦}}$, $\boxed{\text{⑩}}$ if $\boxed{\text{⑦}} < x < \boxed{\text{⑧}}$, and $\boxed{\text{⑪}}$ if $x > \boxed{\text{⑧}}$.

3. A fair die is rolled three times. The die is a standard six-sided die with an equal probability of rolling each of the numbers 1–6. Let a , b , and c denote the numbers we get on the first, second, and third roll, respectively. Put $X = 100a + 10b + c$ and $Y = 100c + 10b + a$.

- (1) The probability that $a < b < c$ is .
- (2) The minimum value of X , which is a multiple of 4, is ①. The probability that X is a multiple of 4 is ②.
- (3) The probability that $X = Y$ is ①, and that $X = Y + 198$ is ②.
- (4) The maximum value of $X - Y$ is .