

2023 年度日本政府 (文部科学省) 奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2023

学科試験 問題

EXAMINATION QUESTIONS

(高等専門学校留学生)

COLLEGE OF TECHNOLOGY STUDENTS

数 学

MATHEMATICS

注意 ☆試験時間は 60 分

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

(2023)

MATHEMATICS

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	
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1 Answer the following questions and write your answers in the boxes provided.

1) Solve the equation $2x^3 + 3x^2 - 1 = 0$.

$$x =$$

2) Solve the equation $4^x - 9 \cdot 2^{x-1} + 2 = 0$.

$$x =$$

3) Express $\left| |2\sqrt{5} - 4| - |\sqrt{5} - 3| \right|$ without the absolute value symbol.

4) Solve the inequality $\log_{10}(x+3) + \log_{\frac{1}{10}} x > 1$.

- 5) Solve the inequality $2 \cos^2 x + \cos x < 0$ ($0 \leq x \leq \pi$).

- 6) Let $\vec{a}$ and $\vec{b}$ be vectors satisfying that the angle formed by them is $\frac{3}{4}\pi$, and $|\vec{a}| = \sqrt{2}$, $|\vec{b}| = 3$. Find the value of $|3\vec{a} + \vec{b}|$.

$|3\vec{a} + \vec{b}| =$

- 7) Suppose that the vector $\vec{x} = (a, b, 1)$ is perpendicular to the plane passing through the three points $(1, -1, -3)$, $(2, 0, -1)$, $(3, -2, 4)$. Find the values of a and b .

$a =$ $b =$

- 8) Find the tangent line to the curve $y = \sqrt{x-4}$ which goes through the point $(0, 0)$.

$y =$

9) Calculate $\sum_{k=1}^{10} (1 + 2 + 3 + \cdots + k)$.

10) Calculate $\lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 2 \sin x}{1 - \sqrt{3} \tan x}$.

11) Let $f(x) = \frac{\log_e \sqrt{x}}{x}$. Find the value of $f'(1)$.

$f'(1) =$

12) Calculate $\int_0^1 (x + 1)(2x - 1)^4 dx$.

2 Let $A = \begin{pmatrix} x & x+2 \\ -2 & -3 \end{pmatrix}$ where x is a real number. Answer the following questions and write your answers in the boxes provided.

- 1) When A does not have its inverse A^{-1} , find the value of x .

$$x =$$

- 2) Suppose that there are real numbers α and β satisfying $A^2 = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$. Find the value of x .

$$x =$$

- 3) When the value of x is determined in 2), find the inverse A^{-1} of A .

$$A^{-1} = \begin{pmatrix} & \\ & \end{pmatrix}$$

- 4) When the value of x is determined in 2), calculate $A^{11} + A^{12} + A^{13}$.

$$\begin{pmatrix} & \\ & \end{pmatrix}$$

3 Let $I_n = \int_0^{\frac{\pi}{2}} \frac{\sin 2x (\sin x)^{2n-1}}{2n-1} dx$ and $S_n = \sum_{k=1}^n I_k$ where n is a natural number. Answer the following questions and write your answers in the boxes provided.

1) Calculate I_1 .

$$I_1 =$$

2) Calculate I_n and express it in terms of n .

$$I_n =$$

3) Calculate S_n and express it in terms of n .

$$S_n =$$

4) Calculate $\lim_{n \rightarrow \infty} S_n$.

$$\lim_{n \rightarrow \infty} S_n =$$