

2024 年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR THE JAPANESE
GOVERNMENT (MEXT) SCHOLARSHIP 2024

学科試験 問題

EXAMINATION QUESTIONS

(学部留学生 (文科系専攻))

UNDERGRADUATE STUDENTS

数 学 (A)

MATHEMATICS(A)

注意 ☆試験時間は **60 分**。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	
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1. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) Let $0 < a < 1$. If $\sqrt{a+1+2\sqrt{a}} - \sqrt{a+1-2\sqrt{a}} = 1$, then $a = \boxed{[1-1]}$.

(2) Let $x \geq 0$ and $y \geq 0$ be integers which satisfy $2x^2 + 7xy + 3y^2 - 2 = 0$. Then $x = \boxed{[1-2]}$ and $y = \boxed{[1-3]}$.

(3) Suppose the straight line ℓ is tangent to the graph of the quadratic function $y = x^2 - 4x + 7$ and passes through the point $(4, 7)$. Then the equation of ℓ is $y = \boxed{[1-4]}x + \boxed{[1-5]}$.

(4) Let $0^\circ < \theta < 90^\circ$ satisfy $5\theta = 180^\circ$. Then $\sin 2\theta = \boxed{[1-6]}\sin^3 \theta + \boxed{[1-7]}\sin \theta$ and

$$\cos 36^\circ = \frac{\boxed{[1-8]} + \sqrt{\boxed{[1-9]}}}{\boxed{[1-10]}}.$$

(5) If a natural number x satisfies

$$5^8 - 1 = (5^7 + 5^6 + 5^5 + 5^4 + 5^3 + 5^2 + 5^1 + 1)x,$$

then we have $x = \boxed{[1-11]}$. If we divide an septenary number 6532452 by 6, its remainder is $\boxed{[1-12]}$.

(6) Consider several natural numbers whose least common multiples with 18 is three times of their greatest common divisors with 18. Describe such natural numbers in the ascending order by separating them with commas $\boxed{[1-13]}$.

- (7) For an equilateral triangle ABC with an edge length is 2, let D and x ($0 < x < 2$) be a point on edge AB and the length of AD, respectively. Consider the shortest path starting from point D, passing through a point on edge BC and a point on edge CA, and returning to point D. When we express the length of the path by $\sqrt{ax^2 + bx + c}$, we have $a = \boxed{[1-14]}$, $b = \boxed{[1-15]}$, and $c = \boxed{[1-16]}$.
- (8) In a box, there are six cards numbered 1, 2, 3, 5, 6, and 12, a number on each card. We take a card from the box and return it to the box. We repeat the trial three times. The probability that the sum of the three numbers on the cards is 15 is $\boxed{[1-17]}$.
- (9) Consider a straight line that is line symmetric by $y = 3$ to $3x + 2y + 3 = 0$ in the x, y -coordinate system. If line $ax + by + c = 0$ is a line that is line symmetric with the said straight line by line $x = 2$, then we have $a = \boxed{[1-18]}$, $b = \boxed{[1-19]}$, $c = \boxed{[1-20]}$, where a is a non-negative integer, b and c are integers, and a , b , and c are relatively prime integers.

2. Let $a > 0$. Consider the function $f(x) = |x| \left(x^2 - \frac{3}{2}(a+1)x + 3a \right)$.

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet. Fractions should be simplified as much as possible.

(1) The function $f(x)$ takes the minimum

$$\begin{cases} \boxed{[2-1]}a + \boxed{[2-2]}, & 0 < a < \boxed{[2-3]}, \\ \boxed{[2-4]}, & \boxed{[2-3]} \leq a \leq \boxed{[2-5]}, \\ \boxed{[2-6]}a^3 + \boxed{[2-7]}a^2, & a > \boxed{[2-5]}. \end{cases}$$

(2) When $a = \boxed{[2-5]}$, the area of the region enclosed by graph $y = f(x)$ and the x -axis is $\boxed{[2-8]}$.

3. Let the lengths of edge AB, edge BC and diagonal CA of quadrilateral ABCD be 3, 5, and 7, respectively. Let $\angle CDA$ be 60° . The area of triangle BAD is three times larger than the area of triangle BCD. The lengths of edge DA and edge DC are denoted by a and b , respectively. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet. Each answer should be an integer or a fraction. Fractions should be simplified as much as possible and the denominators should be positive integers. The numbers in square roots should be also positive integers.

(1) $\angle ABC$ is $\boxed{[3-1]}^\circ$.

(2) $a = \boxed{[3-2]} b$.

(3) $b = \boxed{[3-3]} \sqrt{\boxed{[3-4]}}$.

(4) The area of triangle BAD is $\boxed{[3-5]} \sqrt{\boxed{[3-6]}}$.