

2024 年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR THE JAPANESE
GOVERNMENT (MEXT) SCHOLARSHIP 2024

学科試験 問題

EXAMINATION QUESTIONS

(学部留学生 (理科系専攻))

UNDERGRADUATE STUDENTS

数 学 (B)

MATHEMATICS (B)

注意 ☆試験時間は **60 分**。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with your answers.

(1) Compute the following limit.

$$\lim_{t \rightarrow 5} \frac{t - 5}{t^2 - 2t - 15} = \frac{1}{\boxed{}}.$$

(2) Let $\alpha = \sqrt{2} + \sqrt{3}$. Then $\alpha^2 = \boxed{\textcircled{1}}$, and α is the solution of the equation $x^4 - \boxed{\textcircled{2}}x^2 + 1 = 0$ with integer coefficients.

(3) For the equation $\sin x \cos x = -\frac{1}{4}$ in the range $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, the solutions are $x = \boxed{\textcircled{1}}, \boxed{\textcircled{2}}$.

(4) The absolute value of a complex number $x + yi$ is defined by $|x + yi| = \sqrt{x^2 + y^2}$ with the imaginary unit $i = \sqrt{-1}$. Let $\beta = a + ai$ for a positive real number a . If a is $\boxed{\textcircled{1}}$ or $\boxed{\textcircled{2}}$, then there exists a unique complex number z satisfying $|z| = 1$ and $|z - \beta| = 5$.

(5) The product of the two real values of a such that the following simultaneous equations

$$\begin{cases} ax + y + z = 3x \\ x + ay + 2z = 2y \\ ax + ay + 4z = 2x + y \end{cases}$$

have non-trivial solutions (solutions (x, y, z) such that $(x, y, z) \neq (0, 0, 0)$) is $\boxed{}$.

(6) Consider the following set of data:

Data value	45	55	65	75	85
Frequency	3	6	4	2	1

The mean is $\boxed{\textcircled{1}}$ and the population standard deviation is $\sqrt{\boxed{\textcircled{2}}}$.

2. For a real number a , we set $f_a(x) = x^3 + 2ax^2 + 2ax + 1$.

(1) $f_a(x) = 0$ has an integer solution $x = \boxed{}$ regardless of the value of a .

(2) When $f_a(x) = 0$ is solved with solutions that include complex numbers, it has a unique solution if $a = \boxed{\textcircled{1}}$, two solutions if $a = \boxed{\textcircled{2}}$, and otherwise three solutions.

(3) Let α and β be non-integer solutions of $f_{-1}(x) = 0$. If the equation with rational coefficients

$$bx^3 - cx + 1 = 0$$

has solutions α and β , then $b = \boxed{\textcircled{1}}$ and $c = \boxed{\textcircled{2}}$.

3. Consider two ellipses E_1 and E_2 sharing one focus F , and let F_1 be the other focus of E_1 and F_2 be the other focus of E_2 . Let E_1 and E_2 intersect at only one point P . Suppose that the lengths of the minor axes of E_1 and E_2 are 8 and 24, respectively, and let $\overline{F_1F} = 6$, $\overline{F_2F} = 10$.
- (1) If Q_1 denotes one end of the minor axis of E_1 and Q_2 denotes one end of the minor axis of E_2 , then $\overline{Q_1F} = \boxed{\textcircled{1}}$ and $\overline{Q_2F} = \boxed{\textcircled{2}}$.
- (2) The length of the major axis of E_1 is $\boxed{\textcircled{1}}$ and that of E_2 is $\boxed{\textcircled{2}}$.
- (3) $|\overline{F_1P} - \overline{F_2P}| = \boxed{}$.