

2024 年度日本政府 (文部科学省) 奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2024

学科試験 問題

EXAMINATION QUESTIONS

(高等専門学校留学生)

COLLEGE OF TECHNOLOGY STUDENTS

数 学

MATHEMATICS

注意 ☆試験時間は 60 分

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

MATHEMATICS

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	
-------	--

1 Answer the following questions and write your answers in the boxes provided.

1) Solve the equation $x^3 + 4x^2 + 2x - 4 = 0$.

$$x =$$

2) Solve the inequality $4^x - 3 \cdot 2^{x+1} + 8 < 0$.

$$x =$$

3) Solve the equation $\log_2(x+1) + \log_2(x+3) = 3$.

$$x =$$

4) Solve the equation $\sin 2x - \sqrt{2} \cos x = 0$, in the interval $0 \leq x < 2\pi$.

- 5) Find the equation of the tangent having slope 2 to the curve $y = e^{2x}$.

$$y =$$

- 6) Let $f(x) = \left(\sin \frac{x}{2}\right)^3$. Find the derivative value $f'\left(\frac{\pi}{3}\right)$ at $x = \frac{\pi}{3}$.

$$f'\left(\frac{\pi}{3}\right) =$$

- 7) Calculate the integral $\int_1^{e^2} \frac{\log_e x}{x^2} dx$.

- 8) Calculate the sum of the series $\sum_{k=1}^{10} k(3k+1)$.

- 9) Calculate the limit $\lim_{x \rightarrow 0} \frac{\tan^2 2x}{x^2}$.

- 10) Calculate the sum $\sum_{n=1}^{10} a_n$, if the sequence $\{a_n\}$ satisfies

$$a_1 = 4, \quad a_{n+1} = \frac{1}{2}a_n \quad \text{for } n \geq 1.$$

- 11) Find the coordinates of the intersection point of the plane $4x - y + 2z = 7$ and the line $x + 1 = \frac{y + 3}{2} = \frac{z + 8}{3}$.

$x = \quad, y = \quad, z = \quad$

- 12) Find the angle formed by two vectors $\vec{a} = (1, -2, 3)$ and $\vec{b} = (2, 3, -1)$.

2 Suppose that the matrices $A = \begin{pmatrix} -1 & -2 \\ 4 & 5 \end{pmatrix}$, $P = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$ and $D = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ satisfy $AP = PD$, where a and b are real numbers. Then answer the following questions and write your answers in the boxes provided.

1) Find a and b .

$$a = \quad , b =$$

2) Calculate the inverse P^{-1} of P .

$$P^{-1} =$$

3) Calculate A^n , if n is a natural number.

$$A^n = \left(\begin{array}{cc} & \\ & \end{array} \right)$$

3 Let $I_n = \int_0^{\frac{\pi}{3}} \tan^n x \, dx$, where n is a nonnegative integer. Answer the following questions and write your answers in the boxes provided.

1) Calculate I_1 .

$$I_1 =$$

2) Calculate $I_0 + I_2$.

$$I_0 + I_2 =$$

3) Calculate $I_n + I_{n+2}$ and express it in terms of n .

$$I_n + I_{n+2} =$$

4) Calculate I_3 .

$$I_3 =$$