

2025 年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR THE JAPANESE
GOVERNMENT (MEXT) SCHOLARSHIP 2025

学科試験 問題

EXAMINATION QUESTIONS

(学部留学生 (理科系専攻))

UNDERGRADUATE STUDENTS

数 学 (B)

MATHEMATICS(B)

注意 ☆試験時間は 60 分。

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with your answers.

- (1) Among the four values $\frac{7}{8}$, $\frac{\sqrt[4]{10}}{2}$, $\sin \frac{\pi}{3}$ and $\log_8(4\sqrt{2})$, the smallest value is (i) and the largest value is (ii).

- (2) Let $f(x) = x^3 - \frac{x}{2} + 2$. The positive value of c satisfying the mean value theorem

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

on the interval $[a, b] = [-1, 1]$ is .

- (3) The focus of the parabola $4y = x^2$ after translation by -1 unit along the x -axis and by 1 unit along the y -axis is .

- (4) Consider a triangle with vertices A, B and C such that $\overline{AB} = \overline{AC}$. Let $\angle A = \alpha$ and $\angle B = \beta$. If $\tan(\alpha + \beta) = -\frac{5}{4}$, then $\tan \alpha =$.

- (5) Let $\{a_k\}_{k=1,2,\dots}$ be the sequence determined by

$$a_1 = 2,$$

$$2a_k - a_{k-1} = \frac{1}{2^k} \text{ for } k \geq 2.$$

Then $a_3 =$ (i), and the general term is $a_n = \frac{\text{(ii)}}{2^{n+1}}$.

- (6) Consider integer solutions a and b to the equation $a(b + 1) = 2(b + 4)$. The smallest integer b satisfying this equation is (i), and the largest of such integer is (ii).

2. Let $\log$ denote the natural logarithm, so that the base is the Napier's constant e . Consider two curves

$$C_1 : y = \log(2x) + \frac{5}{2}$$

$$C_2 : y = -2x^2 + a,$$

where a is a real number.

- (1) The tangent l to C_1 at $x = \frac{1}{2}$ is $y = \boxed{\text{(i)}}x + \boxed{\text{(ii)}}$. If $a = 3$, then the area of the region bounded by C_2 and l is $\boxed{\text{(iii)}}$.
- (2) If $a = 2 - \frac{1}{2e}$, then C_1 and C_2 have a unique common tangent line

$$y = \frac{2}{\boxed{\text{(i)}}}x + \boxed{\text{(ii)}}.$$

3. Consider two sets

$$A(m) = \{3k + 1 \mid k = 1, \dots, m\},$$

$$B(n) = \{2k \mid k = 1, \dots, n\}$$

for positive integers m and n .

- (1) The largest integer in $A(18) \cap B(25)$ is $\boxed{}$.
- (2) The set $A(44) \cap B(77)$ contains $\boxed{\text{(i)}}$ integers, while $A(44) \cup B(77)$ contains $\boxed{\text{(ii)}}$ integers.
- (3) The range of n such that the sum of the integers in $A(30) \cap B(n)$ is 252 is $\boxed{\text{(i)}} \leq n \leq \boxed{\text{(ii)}}$. In this case, the largest integer in $A(30) \cap B(n)$ is $\boxed{\text{(iii)}}$.